higher frequencies, on the other hand, it has been shown in reference 4, that the major contribution to the conductivity comes from hops between such pairs of impurities, that the hopping rate back and forth between them corresponds to the applied frequency. If the hopping rates are changed by the magnetic field (or by some other cause) the resulting change in conductivity will just be a measure of the change of population of carriers, whose hopping rate corresponds to the applied frequency.

The mechanism of the negative magnetoresistance is not understood. In terms of Mikoshiba's theory,[5] there are two major causes for a magnetoresistance of hopping conduction: the phase effect and the size effect. The phase effect can be immediately discarded, as it must cause a positive magnetoresistance and furthermore must be vanishing for $H\|E$. The size effect could be responsible for the negative magnetoresistance, if one assumes that the magnetic field splits the heavy and light holes raising the latter, so that the relative contribution of the light hole to the localized wave function increases with magnetic field, thus increasing the overlap integrals. It is hard to see, however, how such an explanation could be made consistent with the magnetic field dependence. It also remains to be seen, whether the negative magnetoresistance is characteristic of p-type Si, or whether it is a freak of the boron impurity.

Helpful discussions with T. Holstein are gladly acknowledged. The authors are also indebted to J. Albrecht for his able and extensive help with the apparatus.

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[5] N. Mikoshiba and S. Gonda, Phys. Rev. 127, 1954 (1962).

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Magnetoabsorption and the Band Gap of Bi

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Data are presented to show that the magnetoabsorption of Bi in the infrared is due to direct interband transitions between Landau levels rather than from de Haas–van Alphen type oscillations of the Fermi level as previously hypothesized. These new data permit a more accurate determination of the energy gap and yield a value of 0.024 eV for this energy.

A REINVESTIGATION of the magneto-oscillations in the infrared transmission of Bi observed by Boyle and Rodgers[1] shows that they result from resonant interband transitions between Landau levels of the valence and conduction bands rather than from de Haas–van Alphen type oscillations in the Fermi level as previously hypothesized. These new data extend to lower energies the measurements of interband transitions made by Brown *et al.*[2] using reflection techniques. Because of the nonparabolic shape of the Bi energy bands, this extension permits a more accurate determination of the energy gap. A value of 0.024 eV is obtained in contrast to 0.046 eV obtained by Brown *et al.*

At low temperatures Bi becomes transparent to infrared radiation between approximately 20 and 50μ. Close to the short-wavelength cutoff of this pass band, oscillations in the transmission at fixed wavelength are observed when the magnetic field strength is varied as shown in Fig. 1. These oscillations are periodic in $1/H$. The hypothesis that they arise from a de Haas–van Alphen type variation of the Fermi level requires that their period be independent of photon energy. The data of Fig. 2, which show the position of the transmission minima (in terms of H) vs photon energy, do not satisfy this criterion. Instead, these data are

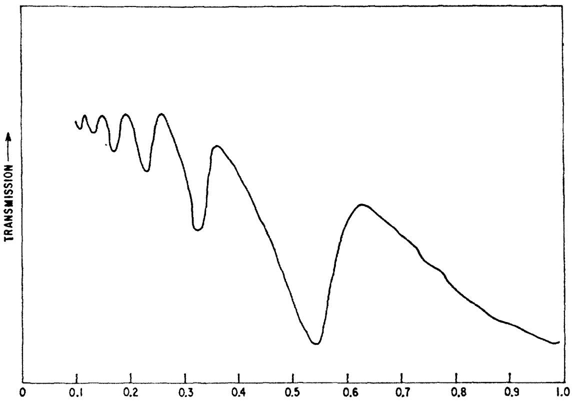

FIG. 1. Magnetic field variation of the transmission of Bi, $H\|$binary axis$\perp$trigonal axis; Poynting vector of the 20.7 μ radiation$\|$trigonal axis; bath temperature 4.2°K. 1 A$\approx$12 kG. Sample thickness$\approx 10\mu$.

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[1] W. S. Boyle and K. F. Rodgers, Phys. Rev. Letters 2, 338 (1959).

[2] R. N. Brown, J. G. Mavroides, M. S. Dresselhaus, and B. Lax, Phys. Rev. Letters 5, 243 (1960).

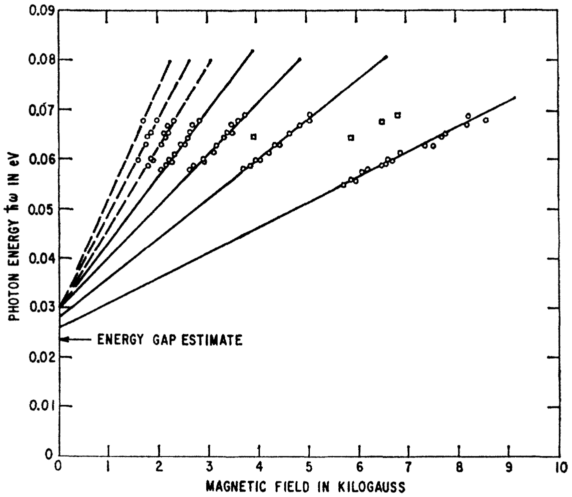

FIG. 2. Position of the transmission minima vs photon energy for Bi, H‖binary axis⊥trigonal axis; Poynting vector of the radiation‖trigonal axis.

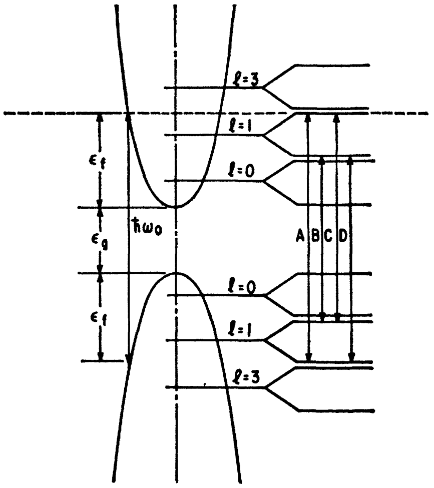

FIG. 3. Band model for Bi in a magnetic field.

consistent with an explanation in terms of interband resonant absorptions. These data are quite similar to those obtained by Brown *et al.*[2] in their interband magnetoreflection experiments at higher photon energies. The transmission data, however, show fewer lines with steeper slopes and lower intercept energies than the reflection data. If the absorptions are numbered and a plot of the reciprocal of their magnetic field position vs this number is made, the result is a straight line with an intercept located at zero provided the line numbers are assigned only the odd integer values. For photon energies very close to the short-wavelength cutoff of the pass band, the line shape of the absorption changes and shoulders having energies corresponding to the even integral values appear.

In a semimetal such as Bi where a second valence band is located below the conduction band having an effective mass equal to that of the conduction band, an absorption edge for direct optical transitions should occur at a photon energy of the gap plus twice the Fermi energy. This transition ($\hbar\omega_0$) is shown in Fig. 3. When a magnetic field is applied, direct transitions between Landau levels subject to the selection rule $\Delta l=0$ can occur. Because the conduction band is partially filled, these transitions are possible only for energies for which the conduction band level has been raised above the Fermi energy. Since the valence band level has been lowered by an equivalent amount, in the absence of spin, the interband magneto-optical transitions should be seen only for energies greater than the absorption edge energy ($\hbar\omega_0$). This was shown by Dresselhaus and Dresselhaus.[3]

If the splitting of the Landau levels due to spin is taken into account, the above restriction is no longer valid and transitions for energies less than that of the absorption edge are permitted. This splitting can be quite large and is equal to the cyclotron energy for materials with very low effective masses such as Bi.[4] Since these strong absorptions can then occur in a region which is otherwise generally transparent, they cause very pronounced effects.

Figure 3 shows the four possible types of transitions, when spin splitting is included, consistent with $\Delta l=0$ labeled A, B, C, and D. Only the transition labeled C can be observed for energies less than the edge energy ($\hbar\omega_0$). The requirement that the upper level be above the Fermi level for this transition yields an energy less than the edge energy by an amount equal to the spin splitting $\hbar(\omega_0-\omega_c)$. For transitions A and B the threshold energy is $\hbar\omega_0$, and for D it is $\hbar(\omega_0+\omega_c)$. When the conduction band and valence band effective masses are equal and the spin splitting is equal to the cyclotron energy, transitions of the type C have an energy given by

$$\hbar\omega=(2l+1)\hbar\omega_c+\epsilon_g,$$

and transitions A and B an energy given by

$$\hbar\omega=2l\hbar\omega_c+\epsilon_g.$$

The odd integer spacing of the observed transition plotted on a $1/H$ vs number plot verifies that the type C transition is indeed responsible for the observed effects. Transitions of the type A and B are responsible for the shoulders of the absorption lines at the highest photon energies. The higher energy magnetoreflection experiments should respond to all of these transitions.

An extension of these transitions to high energies should join the data of Brown *et al.*[2] for this orientation. This cannot be accomplished with a straight-line extrapolation, but requires a curve indicating that the effective mass increases as one moves to higher energies

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[3] M. S. Dresselhaus and G. Dresselhaus, Phys. Rev. **125**, 499 (1962).

[4] M. H. Cohen and E. I. Blount, Phil. Mag. **5**, 115 (1960).

in the band. Such an increase is predicted by the theories of Lax[5] and of Cohen.[6] Both of these theories, however, predict a stronger curvature than is actually observed. Because of the curvature of these lines, a linear extrapolation of the data to zero field yields a value for the energy gap which is too high. This error is reduced when lower energy transitions are used. The absorption data should then yield a better estimate of the energy gap than the higher energy reflection data. A systematic increase of the intercept value for transitions of higher quantum number can be seen in both sets of data. An estimate of the actual energy gap can be obtained by extrapolating these values to zero number. This yields a value of 0.024 eV. A similar estimate of the effective mass at the band minima for this orientation yields a value of 0.0065 m_0 as compared to a value of 0.01 m_0 for this mass when measured at the Fermi surface.

This value of 0.024 eV for the edge energy and the value for the Fermi energy of 0.022 eV[7] yields 0.068 eV for the position of the optical absorption edge $(\hbar\omega_0)$ which is in reasonable agreement with the observed transmission of the sample.

The fact that the even transitions are not observed for energies below the absorption edge indicates that the selection rule $\Delta l=0$ is obeyed. This implies that the valence band is located directly below the conduction band,[8] which lends further support to the current contention that only three conduction band ellipsoids exist.[9]

The material used for this investigation came from two crystals of Bi, one grown by pulling from the melt, and the other grown by a horizontal zone technique. Both showed identical magneto-optical effects. One end of the samples was mounted onto the cold finger of an optical cryostat in order to minimize strain. An enclosure filled with helium as an exchange gas was used to keep the sample temperature close to the bath temperature.

The author would like to thank Professor H. Levinstein of Syracuse University for supplying the Cd-doped Ge used in the construction of the infrared detector for these experiments and Dr. D. Brown and M. L. Keifer of this laboratory for supplying the Bi crystals. I am also indebted to Dr. F. Ham and Dr. H. Ehrenreich for their help in preparing this paper.

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[5] B. Lax, J. G. Mavroides, H. J. Zeiger, and R. J. Keyes, Phys. Rev. Letters **5**, 241 (1960).

[6] M. H. Cohen, Phys. Rev. **121**, 387 (1961).

[7] D. Weiner, Phys. Rev. **125**, 1226 (1962).

[8] J. G. Mavroides and D. Dresselhaus, in *The Fermi Surface*, edited by W. A. Harrison and M. B. Webb (John Wiley & Sons, Inc., New York, 1960), pp. 210, 211.

[9] A. L. Jain and S. H. Koenig, Phys. Rev. **127**, 442 (1962).

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Lifetime of *d*-Band Holes in InSb[†]

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Profiles of the In $L\beta_2$ x-ray emission in metallic indium and in InSb have been recorded by means of a vacuum two-crystal instrument. According to the usual term assignments, the final state for this transition contains a hole in the 4*d* band. With suitable assumptions regarding both this state and the initial state, an approximate measure of the decay width of the 4*d* hole is obtained.

RECENT analysis of the optical properties of 3–5 semiconductors[1] has demonstrated the effect of interband absorption from the *d* bands of several of these semiconductors. In order to obtain a satisfactory fit with the data, the final state for optical absorption must be assigned an extremely short relaxation time.[1a] This final state may be considered to include a *d*-band hole and a "conduction band" electron; the coupling of these may, of course, be significant. The required short relaxation time may, in principle, arise from (1) a short lifetime for the *d*-band hole, or (2) a short lifetime for the electron state.

It is the purpose of this note to report the results of a measurement which tends to exclude the first alternative above. This measurement is of the line shape of the indium $L\beta_2$ x-ray transition. The initial state for this transition includes a hole in the In 2*p* band (L_{III} state)

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[1] H. R. Philipp and H. Ehrenreich, Phys. Rev. Letters **8**, 92–94 (1962).

[1a] *Note added in proof.* In reference 1, the broad optical structure was discussed in terms of lifetime broadening of an otherwise narrow interband transition. This report is written accordingly. It was brought to the author's attention that, in more recent work, other possibilities have been considered by Philip and Ehrenreich (Physical Review, this issue). They now consider also the possibility that the broad optical structure may arise from superposition of transitions at different points in the zone. Such a view is not inconsistent with the results reported here.